Exercise 60

Find the derivative of the function. Simplify where possible.

$$y = \arctan \sqrt{\frac{1-x}{1+x}}$$

Solution

Use the quotient rule, the chain rule, and the derivatives of the inverse trigonometric functions listed on page 214.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx} \arctan \sqrt{\frac{1-x}{1+x}} \\
&= \frac{1}{1+\left(\sqrt{\frac{1-x}{1+x}}\right)^2} \cdot \frac{d}{dx}\left(\sqrt{\frac{1-x}{1+x}}\right) \\
&= \frac{1}{1+\frac{1-x}{1+x}} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{d}{dx}\left(\frac{1-x}{1+x}\right) \\
&= \frac{1}{1+\frac{1-x}{1+x}} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{\left[\frac{d}{dx}(1-x)\right](1+x) - (1-x)\left[\frac{d}{dx}(1+x)\right]}{(1+x)^2} \\
&= \frac{1}{\left(1+\frac{1-x}{1+x}\right)(1+x)} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{(-1)(1+x) - (1-x)(1)}{(1+x)} \\
&= \frac{1}{(1+x)+(1-x)} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{-1-x-1+x}{1+x} \\
&= \frac{1}{2} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{-2}{1+x} \\
&= -\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{1}{1+x} \\
&= -\frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2} \cdot \frac{1}{1+x}
\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= -\frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{1}{1+x} \\ &= -\frac{1}{2\sqrt{1-x}\sqrt{1+x}} \\ &= -\frac{1}{2\sqrt{(1-x)(1+x)}}.\end{aligned}$$